Featured Lendület Researcher: Péter Pál Pach

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Péter Pál Pach, Senior Research Fellow at the Alfréd Rényi Institute of Mathematics and Head of the MTA-HUN-REN RI Momentum (Lendület) Arithmetic Combinatorics Research Group, continues his research work. Together with his co-authors, he was awarded the Frontiers of Science Award in 2024 for their 2016 paper, in which they developed a new version of the polynomial method. The research group's work focuses on fundamental research in the field of arithmetic combinatorics. However, this field has numerous connections to other branches of mathematics, so the research group's results will also be of use to mathematicians researching many other problems.

Pach, Senior Research Fellow at the Alfréd Rényi Institute of Mathematics, has won support from the Hungarian Academy of Sciences' Momentum Programme more than once, as his Arithmetic Combinatorics Research Group has now been re-established in the Momentum Programme's advanced category. Arithmetic combinatorics is a theoretical branch of mathematics, and, as its name suggests, lies at the intersection of number theory and combinatorics, while also being related to many other areas of mathematics, such as harmonic analysis.

A paper recognized with the Frontiers of Science Award in 2024.

"Within arithmetic combinatorics, we focus primarily on the development of algebraic methods. These techniques are usually referred to as polynomial methods,

which we use to search for patterns, structures and correlations within given sets,"

said Pach. "In some of our studies, we start from a given arithmetic configuration in which we stipulate that it must not occur in a certain set, for example, in a vector space over a finite field. Such a configuration could be, for instance, an arithmetic sequence. The methods we have developed make it possible to study the structure of these sets mathematically."

The research group uses similar linear-algebraic methods in the study of the Alon–Jaeger–Tarsi conjecture and the additive basis conjecture. The Alon–Jaeger–Tarsi conjecture states that if we consider a vector space over a field of at least four elements and take two of its bases, then there always exists a vector that has no zero coordinates with respect to either basis. It was already shown in the late 1980s that the conjecture holds when the size of the field is a proper prime power, that is, a power of a prime with an exponent greater than one. After this, the only case that remained unproven was when the size of the field is prime (rather than a prime power).

During the previous Momentum grant period, Pach and János Nagy resolved this remaining case for sufficiently large primes. To solve the problem, they made use of group ring identities.

"With these algebraic techniques, the first step is always to express the statement in the form of the validity or invalidity of some equation. But the interesting part comes after that," continued the research group leader. "In the case of problems involving arithmetic sequences, we had to develop a new type of rank concept for hypermatrices in order to make progress, whereas for the Alon–Jaeger–Tarsi conjecture, group ring identities provided the key to the solution. Since the conjecture remains open for small primes, we will try to make further progress in this area in the future. This is also important because several other mathematical subfields are closely related to this conjecture.”

One such problem is the additive basis conjecture. This conjecture states that for every prime *p* there exists a constant *c* (which may depend on *p*) such that if we take the union, as a multiset, of *c* bases of a vector space over the field with *p* elements, then this forms an additive basis. This means that by taking linear combinations of the elements of this multiset with coefficients 0 or 1, we obtain all elements of the space. Pach is, of course, well aware that his research topic is difficult for non-mathematicians to understand. He summarised the essence of his group’s research in a slightly more accessible way:

“So we conduct fundamental research in theoretical mathematics. We select the questions we investigate based on how exciting they are, how many other areas of mathematics they connect to, and what potential applications they may have.

The results can be applied in many areas of mathematics. One of the central questions in number theory, for example, is how many numbers can be selected from a set in such a way that they do not contain any long arithmetic sequences. One way of modelling this problem is to prohibit such configurations not among the integers, but in a vector space over a finite field. Since there are methods that make it slightly easier to work in such vector spaces, results obtained there can often be transferred to the setting of the integers as well.

Another motivation for their research stems from the connection between their results and finite geometry. Some of the problems studied in additive combinatorics correspond to certain questions in finite geometry. “Due to the nature of our methods, we are also closely connected to algebra, and these questions are also related to problems in computer science, information theory and graph theory as well,” said the researcher. “Some of our results related to arithmetic sequences are also connected to efforts to speed up matrix multiplication. And matrix multiplication regularly appears in everyday computing applications. However, we do not study these applications directly; rather, others use our theoretical results in their own applied research.”

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